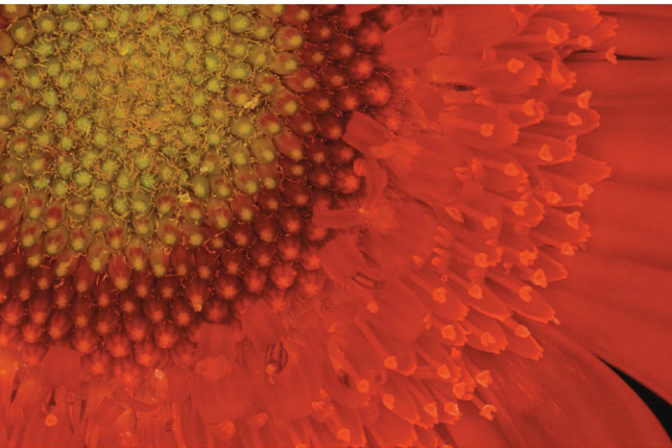


Mathematical Thinking AND Quantitative Reasoning



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Mathematical Thinking and Quantitative Reasoning

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CONTENTS

APPLICATIONS

Blood types 43 Color mixing 50
Counting problems 17, 18, 19, 20,
27, 28, 49, 54, 58, 59 Game
strategies 1, 11, 15 Logical
reasoning 2, 3, 4, 6, 8, 9, 13, 14, 20,
21, 22, 23, 24, 28, 29, 30, 31, 32, 54,
55, 56, 58 Map coloring 14, 55
Mathematics 24, 32 Physics 12,
15 Polya's Problem-Solving
Strategy 17, 18, 19, 20, 21, 22, 28
Reading and interpreting graphs
26, 30, 32, 48, 56, 59 Surveys 44,
45, 51, 52, 57, 58, 59, 60
Voting 46, 47, 49

CHAPTER

1

Problem Solving 1

Section 1.1

Inductive and Deductive Reasoning 2

Investigation: The Game of *Sprouts* by John H. Conway 10

Section 1.2

Problem-Solving Strategies 15

Investigation: Routes on a Probability Demonstrator 27

Section 1.3

Problem Solving Using Sets 33

Investigation: Voting Systems 46

Chapter Summary 53 • Review Exercises 54 • Chapter Test 58

APPLICATIONS

Calculator program 87 Fallacies
105, 108 Logic gates 92, 93, 94,
95 Logic puzzles 111, 114, 116
Mathematics 84 Switching
networks 69, 70, 80, 81, 82
Validity of an argument 99, 100,
101, 102, 103, 104, 107, 108, 109,
110, 111, 112, 115, 116, 119, 120
Warning circuits 70, 71, 82

CHAPTER

2

Logic and Its Applications 61

Section 2.1

Logic Statements and Quantifiers 62

Investigation: Switching Networks 69

Section 2.2

Truth Tables and Applications 73

Investigation: Switching Networks—Part II 80

Section 2.3

The Conditional and Related Statements 84

Investigation: Logic Gates 92

Section 2.4

Arguments 97

Investigation: Fallacies 105

Section 2.5

Euler Diagrams 108

Investigation: Using Logic to Solve Puzzles 113

Chapter Summary 117 • Review Exercises 118 • Chapter Test 120

APPLICATIONS

Automobiles 135, 136, 137, 148, 156, 163, 165, 171, 175, 181, 185, 190, 191, 196, 199 **Business** 129, 138, 149, 156, 157, 163, 170, 182, 186, 191, 196, 197, 198, 200, 201 **College** 136, 138, 144, 145, 148, 155, 166, 196 **Compensation** 138, 140, 141, 144, 152, 165, 166, 180, 190, 198 **Computers** 136, 138, 196 **Death rates** 149, 150, 177 **Gardening** 156, 197, 200 **Government** 156, 158, 164, 169, 180, 197 **Health and medicine** 133, 154, 156, 157, 175, 198 **Housing** 156, 163, 167, 196 **Human behavior** 128, 136, 153, 157, 158, 175, 176, 178, 200 **Measurement** 135, 148, 152, 156, 191 **Money** 129, 137, 138, 140, 141, 142, 143, 152, 154, 155, 156, 165, 166, 173, 174, 178, 191, 199 **Pets** 175, 201 **Populations** 129, 130, 153, 178, 181, 196, 197, 198 **Recreation** 135, 143, 164, 175, 190, 199 **Science** 128, 136, 156, 184, 187, 188, 189, 190, 191, 192, 193, 195, 199, 201 **Space vehicles** 137, 152 **Sports** 135, 136, 151, 152, 174, 198, 200 **Work** 138, 175, 177, 179, 180, 190, 201

CHAPTER**3****Algebraic Models 121****Section 3.1 First-Degree Equations and Formulas 122****Investigation:** Body Mass Index 132**Section 3.2 Rate, Ratio, and Proportion 139****Investigation:** Earned Run Average 151**Section 3.3 Percent 159****Investigation:** Federal Income Tax 171**Section 3.4 Direct and Inverse Variation 183****Investigation:** Gears and Pulleys 188**Chapter Summary 194 • Review Exercises 195 • Chapter Test 199****APPLICATIONS**

Aquariums 216, 289, 290 **Arts and crafts** 210, 243, 246, 250, 253, 256, 259, 260, 262, 308 **Business** 210, 211, 220 **Carpentry** 210, 262 **Construction** 211, 251, 254, 260, 291, 302 **Consumerism** 210, 211, 220 **Energy** 207, 208, 218, 219, 252 **Food** 220, 262, 285, 290, 308, 310 **Gardens** 246, 259, 260, 261, 262 **Geometry** throughout the chapter **Homes and home maintenance** 215, 220, 245, 251, 252, 254, 260, 261, 262, 276, 277, 292, 301, 302, 308 **Land** 220, 261, 262, 308 **Mechanics** 276 **Metallurgy** 244, 253, 291, 292 **Monuments** 289 **Paint** 290, 291, 308 **Parks and recreation** 244, 260, 261, 291, 292, 308 **Satellites** 262 **Science** 210, 211, 220, 221, 262, 287, 288 **Sports and fitness** 210, 215, 220, 221, 245, 246, 250, 259, 260, 261, 262, 263, 290, 301, 308

CHAPTER**4****Measurement and Geometric Models 202****Section 4.1 The Metric System 203****Investigation:** Electrical Energy 207**Section 4.2 The U.S. Customary System 211****Investigation:** Energy 218**Section 4.3 Basic Concepts of Euclidean Geometry 221****Investigation:** Preparing a Circle Graph 234**Section 4.4 Perimeter and Area of Plane Figures 241****Investigation:** Slicing Polygons into Triangles 257**Section 4.5 Properties of Triangles 265****Investigation:** Topology 271

Storage 260, 282, 291 Surveying
275, 309, 310 Travel 276, 301, 302
Trigonometry see Section 4.7

Section 4.6 Volume and Surface Area 277

Investigation: Water Displacement 286

Section 4.7 Introduction to Trigonometry 292

Investigation: Approximating the Values of Trigonometric Functions 299

Chapter Summary 305 • Review Exercises 307 • Chapter Test 309

APPLICATIONS

Aviation 327, 348 Business 334,
339, 344, 345, 364, 365, 366
Compensation 345 Construc-
tion 336, 345 Education 346, 362
Farming 367 Forestry 322
Geometry 318, 321, 322, 323, 337,
364 Health 311, 335, 340, 341,
346, 358, 359, 361, 365, 367
Internet 366 Money 334, 335,
342, 346, 361, 366 Popula-
tions 367 Recreation 321, 324,
327, 330, 341, 344, 348, 367
Science 322, 323, 332, 334, 336,
339, 347, 351, 352, 355, 359, 364,
365 Sports 322, 323, 335, 351
Telecommunications 361, 365
Temperature 330, 361
Vehicles 338, 347, 360, 362

CHAPTER

5

Linear Models 311

Section 5.1 Rectangular Coordinates and Functions 312

Investigation: Dilations of a Geometric Figure 320

Section 5.2 Properties of Linear Functions 324

Investigation: Negative Velocity 332

Section 5.3 Finding Linear Models 337

Investigation: A Linear Business Model 344

Section 5.4 Linear Regression and Correlation 349

Investigation: An Application of Linear Regression 355

Chapter Summary 363 • Review Exercises 364 • Chapter Test 367

APPLICATIONS

Astronomy 410, 413 Automotive
technology 381, 396 Business
377, 380, 382, 383, 412 Chemistry
409, 412, 414 Construction 382
Earth science 396, 409, 410, 414
Engineering 380 Finance 396,
413 Fountains 381 Geometry
375, 381, 382, 412, 414 Hot-air
balloon 382 Manufacturing 380,
413 Meteorology 397 Music
396 Oil supply 410 Physics 380,
383, 396, 397, 412, 413, 414
Population 396, 397 Ranching
381 Recreation 381, 382 Space
science 382 Sports 380, 381, 412,
413 Stopping distance 381
Sustainable yield 383 Water
treatment 380 Welding 396

CHAPTER

6

Nonlinear Models 368

Section 6.1 Introduction to Nonlinear Functions 369

Investigation: Reflective Properties of a Parabola 378

Section 6.2 Exponential Functions 386

Investigation: Chess and Exponential Functions 394

Section 6.3 Logarithmic Functions 398

Investigation: Benford's Law 407

Chapter Summary 411 • Review Exercises 412 • Chapter Test 414

APPLICATIONS

Annual yield 448, 491, 493
 Bonds 471, 474, 476, 491, 492
 Buying on credit 451, 454, 455, 461, 462, 463, 465, 491, 493
 Car leases and purchases 456, 459, 461, 463, 464, 466, 491, 492
 Compound interest 433, 434, 435, 445, 446, 448, 491, 492
 Cost of living 415, 444
 Effective interest rate 441, 447, 491, 493
 Future value 421, 426, 430, 434, 445, 446, 448, 491
 Home ownership 477, 484, 485, 486, 487, 488, 492, 493
 Inflation 438, 439, 447, 491, 493
 Loans 424, 458, 463, 464, 491, 493
 Maturity value 420, 421, 426, 427, 490, 492
 Mortgages 479, 481, 483, 486, 487, 488, 492, 493
 Present value 437, 445, 446, 448, 491, 492
 Simple interest 416, 417, 418, 422, 426, 427, 428, 490, 492
 Stock market 467, 469, 472, 474, 475, 492, 493

CHAPTER**7****The Mathematics of Finance 415**

- Section 7.1** Simple Interest 416
Investigation: Day-of-the-Year Table 423
- Section 7.2** Compound Interest 429
Investigation: Consumer Price Index 443
- Section 7.3** Credit Cards and Consumer Loans 449
Investigation: Leasing Versus Buying a Car 460
- Section 7.4** Stocks, Bonds, and Mutual Funds 467
Investigation: Treasury Bills 473
- Section 7.5** Home Ownership 477
Investigation: Home Ownership Issues 485

Chapter Summary 489 • Review Exercises 490 • Chapter Test 492

APPLICATIONS

Academy awards 536, 537, 549, 561, 579
 Automotive industry 505, 578, 590, 592
 Banking 504, 579
 Business 504, 537, 577, 592
 Communication technology 578
 Commuting 539, 562
 Computers 506
 Construction 505
 Demographics 523
 Dice 495, 510, 526
 Drug testing 519, 525
 Education 505, 506, 524, 537, 561, 562
 Elections 592, 593
 Entertainment 506
 Environment 592
 Genetics 506, 520, 525
 Geometry 504
 Health science 525, 561, 562, 590, 591, 592
 Height distributions 578, 579
 Housing 562, 563, 592
 Human resources 505, 524
 Incomes 562, 577
 Internet 506
 Library science 505
 Lotteries 525, 526, 549, 590
 Meteorology 538, 548
 Monty Hall problem 526
 Physical fitness 561
 Playing cards 502, 510, 513, 523, 524
 Political science 550
 Quality control 505, 591, 592
 Recreation 523, 549
 Restaurants 505, 590
 Roulette 524, 591
 Salaries 523, 563, 577
 Sports 499, 500, 505, 506, 539, 540, 549, 550, 562, 563, 578, 591, 592
 Testing 505, 538, 539, 540, 561, 562, 577, 579
 Weight distributions 549, 578, 579, 592

CHAPTER**8****Probability and Statistics 494**

- Section 8.1** Counting Methods 495
Investigation: Choosing Numbers in Keno 502
- Section 8.2** Introduction to Probability 507
Investigation: Complement of an Event and Sharing Birthdays 521
- Section 8.3** Measures of Central Tendency 527
Investigation: Linear Interpolation and Animation 535
- Section 8.4** Measures of Dispersion 540
Investigation: A Geometric View of Variance and Standard Deviation 547
- Section 8.5** Measures of Relative Position 552
Investigation: Stem-and-Leaf Diagrams 558
- Section 8.6** Normal Distributions 564
Investigation: Cut-Off Scores 576
- Section 8.7** Inferential Statistics 580
Investigation: A Fair Die? 589

Chapter Summary 593 • Review Exercises 595 • Chapter Test 600

APPLICATIONS

Apportionment principle 619
 Business 660, 661, 662, 665 Cell phones 639, 640 Criminal justice 656 Computers 621, 622, 641 Construction 661 Demographics 622 Education 617, 618, 620, 621, 639, 640, 642, 660, 662, 663, 664, 665 Elections 637, 642, 643, 644, 658 Entertainment 638, 639, 641, 643 Family reunion 641 Food science 637, 638 Government 618, 621, 664 Health 619 Management 619, 620, Music 656, 661 Radio stations 640 Recreation 641 Restaurants 638, 639, 645 Scholarships 644, 662 Shapley-Shubik Power Index 657 Social science 621 Sports 639, 657, 663 Technology 661 Transportation 661

CHAPTER**9****Apportionment and Voting 602****Section 9.1****Introduction to Apportionment 603****Investigation:** Apportioning the 1790 House of Representatives 615**Section 9.2****Introduction to Voting 623****Investigation:** Variations of the Borda Count Method 636**Section 9.3****Weighted Voting Systems 646****Investigation:** Blocking Coalitions and the Banzhaf Power Index 654

Chapter Summary 659 • Review Exercises 660 • Chapter Test 664

APPLICATIONS

Architecture 677, 678, 681, 683, 684, 719 Bicycling 675
 Computer networking 695, 701, 721 Machine configuration 694
 Map coloring 702, 703, 704, 705, 712, 715, 716, 721, 724 Overnight delivery 676 Parks 682, 683, 719
 Pen-tracing puzzles 678 Pets 683
 Route planning 684, 694, 698, 700, 702, 721 Scheduling 700, 707, 708, 709, 714, 715, 716, 722, 724 Social networks 681, 722
 Sports 681, 716, 718 Traffic signals 710 Travel 673, 674, 676, 680, 683, 684, 685, 686, 691, 692, 697, 698, 699, 718, 720, 723 World Wide Web 669

CHAPTER**10****The Mathematics of Graphs 666****Section 10.1****Traveling Roads and Visiting Cities 667****Investigation:** Pen-Tracing Puzzles 678**Section 10.2****Efficient Routes 685****Investigation:** Extending the Greedy Algorithm 696**Section 10.3****Map Coloring and Graphs 702****Investigation:** Modeling Traffic Lights with Graphs 710

Chapter Summary 717 • Review Exercises 718 • Chapter Test 722

Solutions to Check Your Progress Problems S1**Answers to Selected Exercises A1****Index I1**

Web Appendix: Algebra Review (Available only online at this textbook's Online Study Center at: hmco.college.com/pic/aufmannMTQR.)

PREFACE

M*athematical Thinking and Quantitative Reasoning* presents an analytical investigation of topics and concepts that are relevant to modern society. Our goal is to demonstrate the power of mathematics and quantitative reasoning in solving contemporary problems.

Mathematical Thinking and Quantitative Reasoning provides glimpses into how mathematics is used to solve real-life problems. Students will learn how prime numbers are used to encrypt information sent across the Internet, the role of modular arithmetic in verifying credit card numbers, how to determine whether to lease or buy a car, how statistics is used to predict the outcome of elections, and how mathematics can be used to evaluate voting systems.

Two features that we have incorporated in the text are Math Matters and Investigations. Math Matters are vignettes of interesting applications related to the topic being discussed. Each section of the text ends with an Investigation, which is an extension of one of the topics presented in that section. For instance, one Investigation extends the ideas of formal logic to logic gates in computers; another Investigation examines how to determine whether a die is fair.

The exercise sets in *Mathematical Thinking and Quantitative Reasoning* have been carefully selected to reinforce and extend the concepts developed in each section. The exercises range from drill-and-practice to interesting challenges. Some of the exercise sets include outlines for further explorations, suggestions for essays, critical thinking problems, and cooperative learning activities. In all cases, the exercises were chosen to illustrate the many facets of the topic under discussion.

The purpose of this text is to strengthen students' quantitative reasoning skills by having them solve a variety of real-world problems. Although we assume that the reader has an intermediate algebra background, each topic is carefully developed, and appropriate material is reviewed whenever necessary. When deciding on the depth of coverage, our singular criterion was to make mathematics accessible.

Student Success

Mathematical Thinking and Quantitative Reasoning is designed to foster student success through an integrated text and media program.

AIM for Success Student Preface

This “how to use this text” preface explains what is required of a student to be successful and how this text has been designed to foster student success. *AIM for Success* can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills.

AIM FOR SUCCESS

Welcome to *Mathematical Thinking and Quantitative Reasoning*. As you begin this course, we know two important facts: (1) You want to succeed. (2) We want you to succeed. In order to accomplish these goals, an effort is required from each of us. For the next few pages, we are going to show you what is required of you to achieve your goal and how we have designed this text to help you succeed.

TAKE NOTE

Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation to swim, but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways that you can benefit from mathematics. But, in the end, the motivation must come from you. On the first day of class it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation. To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out. Do this now.

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate. Although you may not agree that this course should be necessary, it is! If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal of completing your education!

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better. Do this now.

Next to these activities, put the number of hours each week that you spend practicing these activities.

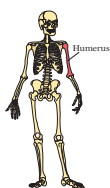
Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend on each activity. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

xvii

page xvii

Applications

In some applications of equations, we are given an equation that can be used to solve the application. This is illustrated in Example 3.



EXAMPLE 3 ■ Solve an Application

Forensic scientists have determined that the equation $H = 2.9L + 78.1$ can be used to approximate the height H , in centimeters, of an adult on the basis of the length L , in centimeters, of the adult's humerus (the bone extending from the shoulder to the elbow).

- Use this equation to approximate the height of an adult whose humerus measures 36 centimeters.
- According to this equation, what is the length of the humerus of an adult whose height is 168 centimeters?

Solution

- Substitute 36 for L in the given equation. Solve the resulting equation for H .

$$\begin{aligned} H &= 2.9L + 78.1 \\ H &= 2.9(36) + 78.1 \\ H &= 104.4 + 78.1 \\ H &= 182.5 \end{aligned}$$

The adult's height is approximately 182.5 centimeters.

- Substitute 168 for H in the given equation. Solve the resulting equation for L .

$$\begin{aligned} H &= 2.9L + 78.1 \\ 168 &= 2.9L + 78.1 \\ 168 - 78.1 &= 2.9L + 78.1 - 78.1 \\ 89.9 &= 2.9L \\ \frac{89.9}{2.9} &= \frac{2.9L}{2.9} \\ 31 &= L \end{aligned}$$

The length of the adult's humerus is approximately 31 centimeters.

CHECK YOUR PROGRESS 3 The amount of garbage generated by each person living in the United States has been increasing and is approximated by the equation $P = 0.05Y - 95$, where P is the number of pounds of garbage generated per person per day and Y is the year.

- Find the amount of garbage generated per person per day in 1990.
- According to the equation, in what year will 5.6 pounds of garbage be generated per person per day?

Solution See page S11.

An Interactive Method

Mathematical Thinking and Quantitative Reasoning is written in a style that encourages the student to interact with the textbook. Each section contains a variety of worked examples. Each example is given a title so that the student can see at a glance the type of problem that is being solved. Most examples include annotations that assist the student in moving from step to step, and the final answer is in color in order to be readily identifiable.

Check Your Progress Exercises

Following each worked example is a *Check Your Progress* exercise for the student to work. By solving this exercise, the student actively practices concepts as they are presented in the text. For each *Check Your Progress* exercise, there is a detailed solution in the Solutions appendix.